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SHORTER COMMUNICATIONS

RADIATION INTERACTION IN NONGRAY BOUNDARY LAYERS

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NOMENCLATURE

A_i,	total band absorptance of the ith band;
A_{0i},	correlation parameter for the ith band;
b_i,	self-broadening coefficient for the ith band;
B_i^2,	correlation parameter for the ith band;
C_{0i}^2,	correlation parameter for the ith band;
e_ω,	Plank's function;
f,	dimensionless stream function $\psi R/\nu\chi\, Gr^{\frac{1}{4}}$;
$f_i(\beta_i)$,	$2{\cdot}94[1 - \exp(-2{\cdot}6\,\beta_i)]$;
Gr,	Grashof number $\dfrac{g\rho^2\beta(T_w - T_\infty)R^3}{\mu^2}$;
k,	thermal conductivity;
n,	pressure broadening exponent;
P,	pressure;
q,	heat-transfer rate;
R,	radius of the cylinder;
u_i, u_i^+, u_i^-,	dimensionless coordinates for the ith band C_{0i}^2Py, $\frac{3}{2}\zeta_i(\eta + t)$, $\frac{3}{2}\zeta_i(\eta - t)$, respectively.

Greek symbols

β_i,	line structure parameters, $B_i^2(b_i\,P/P_0)^n$;
ζ,	interaction parameter, equation (7);
η,	$(y/R)Gr^{\frac{1}{4}}$;
Θ,	$(T - T_\infty)/(T_w - T_\infty)$;
ξ,	interaction parameter, equation (6);
ω,	wave number.

Subscripts

0,	evaluated at one atmosphere;
∞,	free stream;
w,	cylinder wall.

INTRODUCTION

THE GRAY-GAS approximation has been used extensively in the past to study the interaction of gaseous radiation with the other modes of heat transfer. Recently, nongray radiation has received considerable attention [1–4]; however, with but a few exceptions, for example the work of Cess [5], investigations of convection radiation interaction in boundary layers have been restricted to the assumption of a gray gas. The purpose of this note is to examine the behavior and magnitude of nongray radiation interaction in a representative boundary layer situation that could be experimentally created in the laboratory and to discuss the governing parameters. The specific problem treated is the laminar flow of a nongray gas near the lower stagnation point of a heated horizontal cylinder. The cylinder wall is an isothermal gray diffuse surface with a reflectance of one. This surface reflectance does not allow the transport of radiation in the windows of the spectrum.

ANALYSIS

It is assumed that the flow is steady and laminar; except for the buoyancy term, the physical properties of the fluid are assumed to be constant. Limiting the analysis to the stagnation region of an isothermal cylinder and to cylinders of small curvature, the conservation equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{gx(\rho_\infty - \rho)}{R\rho} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where q_r is the spectral radiant flux integrated over all wave numbers ω.

Following previous analyses [1, 2], the radiative term is formulated using the exponential wide-band model of Edwards and Menard [6] and the correlation of Tien and Lowder [7] for the dimensionless band absorption,

$$\bar{A}_i = A_i/A_{0i} = \ln\left\{u_i f_i\left(\frac{u_i + 2}{u_i + 2f_i}\right) + 1\right\}.$$

Introducing the band absorption and the usual transformation, one obtains the following set of conservation equations

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 + \Theta = 0 \tag{4}$$

$$\frac{d^2\Theta}{d\eta^2} + Prf\frac{d\Theta}{d\eta} + \frac{3}{2}\sum_i \xi_i\left\{\int_0^\infty \bar{e}_i\frac{d\Theta}{dt}\bar{A}_i'(u_i^+)\,dt\right.$$

$$-\int_0^\eta \tilde{e}_i \frac{d\Theta}{dt} \bar{A}_i'(u_i^-)\,dt + \int_\eta^\infty \tilde{e}_i \frac{d\Theta}{dt} \bar{A}_i'(-u_i^-)\,dt\Big\} = 0 \quad (5)$$

where

$$\tilde{e}_i = \frac{de_{\omega i}}{dT} \Big/ \left(\frac{de_{\omega i}}{dT}\right)_\infty$$

is evaluated at the center of each band ω_i and $\bar{A}_i'(u_i)$ denotes the derivative of $\bar{A}_i$ with respect to u_i. This derivation assumes that the gas properties are independent of temperature; the properties are evaluated at the free-stream temperature. The exponential kernel approximation [1, 2] has been used coupled with the assumption that the gradient of the Planck function is constant over the band width. Additionally, a black wall at the free-stream temperature is located outside the boundary layer. This latter condition is approximated by a bounding wall in the laboratory. The boundary conditions are

$$\eta = 0: \quad \Theta = 1\cdot0, \quad f = \frac{df}{d\eta} = 0$$

$$\eta \to \infty: \quad \Theta \to 0, \quad \frac{df}{d\eta} \to 0.$$

In addition to the Prandtl number and the derivative of the Planck function $\tilde{e}_i$ the above equations depend on two interaction parameters

$$\xi_i = \frac{R^2 A_{0i} C_{0i}^2 P_0 (de_{\omega i}/dT)_\infty}{k Gr_0^{\frac{1}{4}}} \quad (6)$$

$$\zeta_i = \frac{C_{0i}^2 R P_0}{Gr_0^{\frac{1}{4}}} \left(\frac{P}{P_0}\right)^{\frac{1}{2}}. \quad (7)$$

For the perfectly reflecting wall, the radiative flux at the wall is zero. The total wall heat flux (convective flux) is given by

$$\frac{qR}{k(T_w - T_\infty) Gr^{\frac{1}{4}}} = -\left(\frac{d\Theta}{d\eta}\right)_{\eta=0}. \quad (8)$$

Numerical solutions to equations (4) and (5) were obtained for CO_2, H_2O and CH_4 for a wall temperature ratio $T_w/T_\infty = 1\cdot1$. Since only the behavior of radiation interaction was sought, the Grashof number was based on a cylinder radius of one foot. This stagnation region could be easily created in the laboratory by using a section of a cylinder. For a pressure of one atmosphere, the Grashof number is in the range of 10^7–10^8 for the three gases. The physical properties were obtained from the literature [8–11]. The vibrational–rotational bands used in the heat transfer calculations are H_2O (6·3 μ), CO_2 (15 μ and 4·3 μ) and CH_4 (7·6 μ and 3·3 μ). Of the bands ignored in the analysis, the rotational band of H_2O is the most serious omission; this was not included due to lack of information for the correlation parameters.

RESULTS AND DISCUSSIONS

The heat-transfer results are presented in Fig. 1 as a ratio of the total heat flux with radiation to that for pure convection. The maximum effect of the interaction of radiation occurs at a pressure of less than one atmosphere with the results approaching pure convection as the pressure increases toward infinity or approaches zero. The exponential wide-band model [6] and the band absorptance correlation [7] have not been extensively checked at pressures lower than approximately 0·01 atmosphere, and the agreement between the wide-band model correlation is poor at pressures lower than 0·01 atm ($\beta_i < 0\cdot01$). For pressures below approximately 0·01 atmosphere, the Grashof number is of the order 10^4 or less; therefore, equations (4) and (5) are of questionable validity in this pressure range. In view of the above reasons the results were not extended below $P = 0\cdot01$ atm. Additionally, if the pressures are too low other phenomena such as a change in line shape, slip flow and the breakdown in local thermodynamic equilibrium enter the problem.

For the purpose of discussing the behavior of the heat transfer results, linearized radiation will be assumed; $\tilde{e}_i \to 1\cdot0$ as $T_w/T_\infty \to 1\cdot0$. Treating the high pressure limit first, the dimensionless band absorptance $\bar{A}_i(u_i)$ approaches $\ln(u_i)$ as the pressure approaches infinity ($\beta_i > 1\cdot0$, $u_i \to \infty$). Therefore, the quantity $\sum(\xi_i/\zeta_i)$ is the only radiation parameter appearing in the linearized energy equation. This ratio is indirectly proportional to the square root of the pressure giving an interaction effect which decreases with increasing pressure at high pressures.

At high pressures, the interaction is independent of line structure and the magnitude of $\sum_i \xi_i/\zeta_i$ determines the relative effect of interaction for this boundary-layer problem. The parameter $\sum_i(\xi_i/\zeta_i)\, Gr^{\frac{1}{4}}/R$ can be used to estimate the relative magnitude of radiation interaction for a particular gas in boundary-layer problems. The ratio $R/Gr^{\frac{1}{4}}$ is a measure of the thickness of the boundary layer for the problem studied here. This dimensional parameter is independent of the particular flow situation and pressure, and reduces to

$$\sum_i (\xi_i/\zeta_i) \frac{Gr^{\frac{1}{4}}}{R} = \sum_i \frac{A_{0i}}{k} \left(\frac{de_{\omega i}}{dT}\right)_\infty. \quad (14)$$

This quantity also governs optically-thick interaction in conduction–radiation problems and is plotted by Cess and Tiwari [4] as a function of temperature for CO, CO_2, CH_4, H_2O, N_2O and NH_3. The relative magnitude of the interaction for the gases given in Fig. 1 at $P > 1$ atm, is correctly predicted by a comparison of the values of $\sum_i(\xi_i/\zeta_i)$.

As the pressure decreases, the band absorptance approaches the limit of non-overlapped lines; thus, the line structure parameter plays an important role in the low-pressure results. For convenience, the equations of Edwards and Menard [6] for A_i will be used in this discussion rather than the Tien and Lowder correlation [7]. At low pressures ($\beta_i < 1.0$), there are three relations for A_i (the logarithmic, square root or linear) depending on the magnitude of A_i. If the linear relation dominates, the temperature distribution would be independent of pressure; this is obviously not the case as is shown in Fig. 1. The logarithmic relation is discarded since it represents the limit of large u_i. If the square-root relation dominates the radiation contribution, the only parameter which appears in the linearized form of equation (8) is $\sum_i (\xi_i/\zeta_i^{\frac{1}{2}})\,(\beta_i)^{\frac{1}{2}}$ which is proportional to $[(P/P_0)(2n-1)/4]$.* Taking into account that for CO_2, H_2O

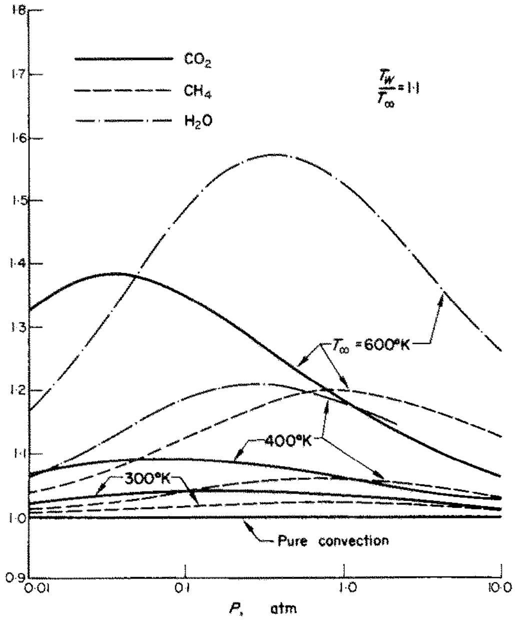

FIG. 1. Total heat transfer.

and CH_4 $1.0 \geq n \geq 0.7$, the effect of radiation interaction approaches zero as the pressure approaches zero. Thus at low pressures the square-root limit of the band absorptance dominates the boundary layer calculations whereas at high pressures the logarithmic limit dominates the calculations. The peak in the heat transfer shown in Fig. 1, is due to the transition between the two limits.

* This pressure dependence is not correct for CO_2 since n varies from 0.65 to 0.8 for the CO_2 band [11]. For purposes of comparison, the assumption of a constant n for the CO_2 bands will be used in this discussion.

A comparison of the values of $\sum_i \xi_i\,(\beta_i/\zeta_i)^{\frac{1}{2}}$ for CO_2, CH_4 and H_2O at a given pressure does not correctly predict the relative magnitude of radiation interaction for the low pressure range in Fig. 1. Carbon dioxide has a much larger relative interaction than that predicted by the magnitude of $\sum_i \xi_i\,(\beta_i/\zeta_i)^{\frac{1}{2}}$. This is understood if it is remembered that two parameters, β_i and ζ_i, govern the change from the logarithmic limit to the square-root relation for a particular band. The 4.3 μ band of CO_2 has a value of ζ_i which is almost an order of magnitude greater than any other band considered. Thus, the transition of $\bar{A}_i(u_i)$ for CO_2 from a logarithmic variation, where $\bar{A}_i'(u_i)$ is inversely proportional to the square root of the pressure, to a square-root variation, where $\bar{A}_i'(u_i)$ is directly proportional to some positive power of the pressure is delayed giving CO_2 a larger interaction than CH_4 or H_2O in the low pressure range shown in Fig. 1. This conclusion is also supported by noting the large shift of the CO_2 interaction peak in Fig. 1 toward lower pressures as the temperature increases. As the temperature increases, the 4.3 μ band increases its role in the interaction behavior due to the shift in the Planck function.

Although the present heat transfer investigation is concerned with a very specific problem in free convection, the results do illustrate, at least qualitatively, the interaction of thermal radiation with convection in boundary layers. Using the high-pressure interaction parameter, the relative magnitude of the effect of radiation interaction on a temperature distribution can be established for pressures greater than one atmosphere. As a final comment, it should be emphasized that additional work is needed in establishing the missing radiative properties of gases, especially the line structure parameter. At the present time, the radiation correlation parameters necessary for the nongray model used here are completely known for only a few common gases.

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DIRECT CONTACT HEAT TRANSFER BETWEEN TWO IMMISCIBLE PHASES DURING DROP FORMATION

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NOMENCLATURE

A, surface area [cm^2];
C', constant;
C_p, specific heat [cal/g °C];
C_{pc}, specific heat of the continuous phase [cal./g °C];
C_{pd}, specific heat of the dispersed phase [cal./g °C];
H, total heat content of the drop at any instant [cal];
M, mass of the drop at any instant, [g]; virtual mass of the drop [g];
P, constant;
Q, volumetric rate of flow of the disperesed phase [cm^3/s];
R, R', constants;
R_c, capillary radius [cm];
S, S', constants;
T, temperature; difference (ΔT) between the temperature of the (internally mixed) forming drop and the temperature of the continuous phase at any instant [°C];
T_1, initial temperature difference ($T_i - T_c$) [deg C];
V, drop volume [cm^3];
a, b, c, coefficients in the polynomials;
d, drop diameter [cm];
g, acceleration due to gravity [cm/s^2];
h, heat-transfer coefficient [cal./s cm^2 °C];
k, thermal conductivity [cal. cm/s cm^2 °C];
m, mass rate of entry of the dispersed phase [g/s];
r, drop radius [cm];
t, time [s];
v_e, velocity of expansion (of the drop) in the first stage of formation [cm/s];
v_r, velocity of rise (of the drop) in the detachment period [cm/s];
x, y, coordinates.

Greek symbols
ρ, density [g/cm^3];
$\Delta\rho$, density difference ($\rho_c - \rho_d$) [g/cm^3];
μ, viscosity [P, c P];
ν, interfacial tension [dyn/cm].

Subscripts
s, of static drop;
c, continuous phase;
d, dispersed phase;
e, expansion stage;
f, formation stage;
i, inlet drop;
o, outlet drop;
r, of rising drop.

Dimensionless groups
N_{Nu}, nusselt number (hd/k);
N_{Re}, reynolds number ($dv\rho/\mu$);
N_{Pr}, prandtl number ($C_p\mu/k$).

Investigations on mass transfer in systems involving a fluid sphere and a liquid continuous medium have normally indicated a considerably large proportion of transfer in the region of formation. The observations on mass transfer